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SHORTER COMMUNICATIONS

DISCUSSION ON THE EFFECT OF TRANSVERSE MASS FLOW ON HEAT TRANSFER AND FRICTION DRAG IN A TURBULENT FLOW OF COMPRESSIBLE GAS ALONG AN ARBITRARILY SHAPED SURFACE

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THE above-mentioned paper [1] deals with measurements of skin friction and heat transfer to a porous flat plate with the injection of freon-12, carbon dioxide, air, and helium into a turbulent boundary layer. The free stream was air, with and without pressure gradient. The paper thus treats many topics. Unfortunately, the presentation suffers from the following inadequacies:

(1). Although the "equilibrium temperature" was mentioned in the paper in connection with heat transfer, the method of determining it, and the results, were not reported. Due to the coupling between heat and mass transfer, the injection of gases different from air strongly affects the equilibrium temperature, particularly when the molecular weight of the injected gas is much different from that of air. In [2] for example, the measured equilibrium temperature with helium injection was up to 40 degF higher than the equilibrium temperature with zero injection. The equilibrium temperature with freon-12 or carbon dioxide injection is not available in the open literature to date to writer's knowledge, and if reported in [1] would have added considerably to its value.

Besides its pertinence to considerations of the coupling between heat and mass transfer, a knowledge of the equilibrium temperature is essential for calculating heat-transfer rates. Consequently, the results presented in [1] are not sufficient to determine the wall heat flux with the injection of gases different from air.

(2). The authors [1] did not compare the results of their measurements with any available previous measurements or theoretical predictions, except in the case of friction factors with air injection and zero pressure gradient. Examples of pertinent previous papers are [2] and some of the references quoted there in connexion with heat transfer with helium injection and zero pressure gradient; [3] and some of the references quoted there in connexion with heat transfer with air injection and zero pressure gradient; and Van Driest's paper referred to in [1] itself.

Some comparison will now be made. The ratio of Stanton number with air injection and zero pressure gradient to that without injection reported in [1] agrees satisfactorily with [3]. The same ratio for helium injection agrees satisfactorily with [2] if the abscissa variable of Fig. 4 [1] is changed from $(c_{pw}\rho_w v_w / c_{p1}\rho_1 u_1 St_o)$ to $(\rho_w v_w / \rho_1 u_1 St_o)$. Without such change, [1] and [2] would disagree markedly.

(3). The authors [1] did not indicate how their porous plate was fabricated, and so the degree of its surface roughness is unknown. Such surface roughness affects skin friction, as for example in [4], where a woven wire porous surface increased the skin friction by about 15 per cent in comparison with a smooth surface, for zero injection. An approximately equal increase for air injection may be inferred from Fig. 8a of [4].

Apparently there must be an error in drawing curve No. 14 in Fig. 2 of [1]. It is much higher than the data in the original reference.

The ratio of skin friction with air injection and zero pressure gradient to that with zero injection reported in [1] is appreciably less than the ratio reported in [4]. At an abscissa value of 1·2, the discrepancy is about 30 per cent.

(4). With the notation of [1] equation (11) in [1] for the Stanton number should read:

$$St = \frac{\rho_w v_w (T_w - T_2)}{\rho_1 u_1 (T_e - T_w)} \times \frac{\text{specific heat of injected gas}}{\text{specific heat of air}}.$$

(5). In [1], it is stated that potential flow velocities ranged from 25 to 75 m/s. The free stream Mach number under these conditions would be too low to show any compressibility effects. Therefore the word "compressible" should be deleted from the title.

In conclusion, the discussed paper appears to be incomplete because of (i) the omission of any data on the equilibrium temperature; (ii) the omission of any comparison with previous measurements or theory except in the special case of friction factors with air injection and zero pressure gradient; (iii) the omission of any specification of surface roughness of plate. Moreover, there is a question regarding the validity of the expression for Stanton number, equation (11), and the abscissa variable in Fig. 4.

REFERENCES

1. P. N. Romanenko and V. N. Kharchenko, The effect of transverse mass flow on heat transfer and friction drag in a turbulent flow of compressible gas along an arbitrarily shaped surface, *Int. J. Heat Mass Transfer* 6, 727–738 (1963).
2. O. E. Tewfik, E. R. G. Eckert and C. J. Shirtliffe, Thermal diffusion effects on energy transfer in a turbulent boundary layer with helium injection. Proceedings of the 1962 Heat Transfer and Fluid Mechanics Institute, Seattle, 42–61 (1962).
3. O. E. Tewfik, E. R. G. Eckert and L. S. Jurewicz, Measurements of heat transfer from a cylinder with air injection into a turbulent boundary layer. *ASME* paper No. 63-HT-45, presented at the *ASME-AIchE* Heat Transfer Conference and Exhibition, Boston (1963).
4. O. E. Tewfik, Some characteristics of the turbulent boundary layer with air injection, *AIAA J.* 1, 1306–1312 (1963).

A COMMENT ON TURBULENT MOMENTUM DIFFUSIVITY WITHIN A CIRCULAR TUBE*

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While Hooper's [1] determination of the mixing length and eddy viscosity from a simple model of the eddy structure is interesting, further discussion of the points marked experimental in Figs. 2 and 3 is needed. It will be shown that the calculation of these values from the experimental velocity distribution involves a supplementary assumption, and that the two sets of values are based on inconsistent assumptions.

The expressions for the mixing length and eddy viscosity can be written as:

$$\frac{l_m}{R} = \left(\frac{r}{R}\right)^{1/2} \div \frac{\mathrm{d}W_d}{\mathrm{d}\,(r/R)} \tag{1}$$

and

$$\frac{\epsilon}{W_\tau R} = \frac{r}{R} \div \frac{\mathrm{d}W_d}{\mathrm{d}\,(r/R)}. \tag{2}$$

The symbols are defined in [1], except for W_d, the velocity defect which is $(W_{\text{center}} - W) \div W_\tau$.

In order to determine the "experimental" points, it is necessary to differentiate the velocity distribution measured at discrete points. Any numerical differentiation procedure depends on an assumption about the form of the functional relation involved and the values of the derivative can be quite sensitive to the assumption. In the present case this is particularly important because both (1) and (2) are indeterminate as $r/R \to 0$.

It is easily seen that the mixing length only has a finite value on the pipe axis if $W \sim (r/R)^{3/2}$ for small values. If the exponent is less than 3/2, the mixing length approaches zero on the axis, if it is greater the mixing length becomes infinite. This was pointed out by Prandtl [2] in 1925, and undoubtedly used by Nikuradze in evaluating the derivative.

On the other hand, the eddy viscosity is finite on the axis only for $W_d \sim (r/R)^2$. Therefore, the finite values on the axis in Figs. 2 and 3 are inherently inconsistent.

In Table 1, the values of W, r, $W_c - W/r^{3/2}$, and $W_c - W/r^2$ are tabulated for a typical Nikuradze [3] traverse and for one reported by Stanton [4]. The indeterminacy close to the axis is clearly shown, and it would appear that the r^2 assumption (i.e. finite eddy. viscosity) is more plausible than the $r^{3/2}$ assumption Furthermore, as the tabulation indicates, the distribution is parabolic over most of the pipe it is very difficult to see how Schlichting [5] obtained the variation shown in Fig. 3. The original reference gives no details.

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